learning of this new value (in 1675, or perhaps after Robert Hooke alluded to it in a letter) recalculated the ratio (∗) and found the closer agreement. Legend has it that he grew so excited that he had to ask for a friend to complete the calculation.[80] For whatever reason, he laid aside his work on gravity for many years, until spurred into action by his rival Hooke.

The Newton-Hooke correspondence of 1679

Newton largely withdrew from the scientific community following a very bitter and very public dispute with Hooke in 1672. In 1679, the president of the Royal Society was Christopher Wren, the architect in charge of rebuilding London after the Great Fire. Wren was well aware of the ability of Professor Newton at Cambridge. He asked Hooke, his right-hand man and the Royal Society's secretary, if Hooke might persuade Newton to write something for the Royal Society's journal, the *Philosophical Transactions*. Hooke could be the soul of courtesy when he chose, and wrote Newton a very conciliatory and flattering letter, including in it a guess Hooke had made about the trajectory of an object dropped from high tower, if the earth's gravity increased inversely as the square of the distance from its center. Newton responded to this letter, and suggested an alternative trajectory. Hooke pointed out an error in Newton's calculation, and Newton replied, finally, with the correct answer. This correspondence seems to have fired Newton's creative powers to return to the mystery of gravity. It is probable but not certain that during the next four years Newton established that Kepler's Laws led unambiguously to the inverse-square law; it may be that he was also able to show that the inverse-square law led to the elliptical paths of the planets, but this is not definitely known. Whatever Newton's results, he kept them entirely to himself. On the one hand, Hooke always maintained that Newton had gotten the idea of an inverse square law from him. This is unlikely. At most, Newton took from Hooke the suggestion that planetary motion was a combination of straight line motion compounded with a force toward the sun. Much evidence exists that Newton had determined (from the apple-moon calculation) that the earth's gravity, at least, was likely to be an inverse-square law some thirteen years before Hooke suggested it in his correspondence. On the other hand, Newton might never have returned to gravity without Hooke's spurring him on; for that, posterity surely owes Hooke thanks.

A meeting in London

One Wednesday in January, 1684, a young astronomer named Edmond Halley (1656-1742) came to London to meet with Wren and Hooke (maybe at a coffee house, then the preferred venue for a conversation in town, or perhaps at the Royal Society itself.)[81] The three discussed gravity. Each was well aware of Kepler's Laws, and each felt that gravity was probably described by an inverse-square law. Wren and Halley had been able to derive an inverse-square law from Kepler's Laws under the drastic simplification that the orbits of the planets were *circular*, but neither could handle the far more difficult mathematics of the actual *elliptical* orbits established by Kepler. The simplified calculation goes quickly.

Pretend that the planets travel in circular orbits. Then for any planet, courtesy of Huygens (and Newton's Second Law!)

$$F_{\text{net}} = ma \propto \frac{v^2}{r}$$

For circular motion, $v = 2\pi r/T$. Substitute this expression in for v to obtain

$$F_{\text{net}} \propto \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Now recall Kepler's Third Law;

$$\frac{a^3}{T^2} = K$$

where K is a constant, and a is the semimajor axis of a planet's elliptical orbit. If instead of ellipses the orbits are taken to be circles, then the semimajor axis a is the same thing as the radius r of a circular orbit, and Kepler's Third Law becomes

$$\frac{r^3}{T^2} = K$$

[80] The story was first published by J. Robison in 1804. See W. W. Rouse Ball, *An Essay on Newton's Principia*, p.23. Rouse Ball did not believe it.

[81] Letter of Halley to Newton, June 29, 1686, reprinted in Rouse Ball's *Essay*, p. 162. Cook writes that they talked in the evening after the Royal Society's meeting, on 14 January. (*Edmund Halley: Charting the Heavens and the Seas*, p. 147)

so that

$$\frac{1}{T^2} = \frac{K}{r^3}$$

Substitute this value into the net force equation above, and obtain

$$F_{\text{net}} \propto \frac{4\pi^2 r}{T^2} = 4\pi^2 r \cdot \frac{K}{r^3} = \frac{4\pi^2 K}{r^2}$$

which is, sure enough, an inverse-square law. But the planets describe ellipses, not circles. Despite their best efforts, neither Wren nor Halley had been able to extend their calculations to ellipses, and each admitted this failure to the other two men. Hooke claimed to have done it. Wren, a wealthy man, offered a prize of a book worth forty shillings (perhaps two weeks' wages) to the first person who could get elliptical orbits from the inverse-square law. Hooke replied that he would not reveal his calculation for a while, because, he said, he wanted the other two to appreciate just how difficult the task was. Months went by, and no solution came from Hooke. Halley, despairing, wrote Wren to say that he feared Hooke would not be "as good as his word".

In March, 1684, Halley's father, a wealthy manufacturer, was murdered on the road to London. Settling his father's estate, Halley had occasion, in August 1684, to be in the neighborhood of Cambridge. He decided to visit Newton nearby to ask him about gravity. The men had met before, but did not then know each other well. An account of this meeting was provided by Newton to the French mathematician Abraham de Moivre (1667-1754), who was interviewed many years later by John Conduitt, the husband of Newton's favorite niece, Catherine Barton.[82] According to de Moivre, Halley asked Newton what shape planetary orbits would be if gravity were an inverse-square law. Newton replied at once, an ellipse. Halley, surprised, asked Newton how he knew this. Newton replied that he had calculated it. Halley asked to see the calculation, but Newton couldn't find it (understandable, if the work had been done four years earlier.) Newton then attempted to redo the calculation on the spot, and made a mistake. He then promised to send Halley the calculation. Halley doubtless thought to himself, "I've heard that before," but this time Newton delivered (or near enough) what Hooke had not. In October, a manuscript bearing the title *De motu corporum in gyrum* (*On the motion of bodies in orbit*) was brought to Halley in which the inverse square law was derived from elliptical orbits (this was not the answer to Wren's question, but its converse.) Halley showed the pages to Wren, who begged Halley to convince Newton to write up his research in an expanded form. It took Halley two years of hard work, flattery and consummate diplomacy to keep Newton from abandoning the work, especially after repeated goading from Hooke. Halley served as editor, copy reader, and sounding board; he arranged for the woodcut illustrations. Eventually Halley even paid to have the work printed. The Royal Society was to have done so, but they were nearly bankrupt, after producing an expensive book on fish that nobody bought. Finally, in 1687, the *Principia* showed the world how gravity worked, and what the new law could explain. You still haven't seen the full law. That's next.

Further reading

The early Greek estimates are described well in the old-fashioned (published in 1899) *A Short History of Astronomy* by Arthur Berry, now an inexpensive paperback from Dover Publications. Ptolemy's *Almagest* is available in English, but it is not a casual read.

An entertaining history of the Copernican Revolution will be found in Arthur Koestler's *The Sleepwalkers.* The section dealing with Kepler and Brahe was published separately in a slim volume called *The Watershed.* Two other histories well worth a read are Thomas S. Kuhn's *The Copernican Revolution* and Edward W. Kolb's *Blind Watchers of the Sky.* Kuhn became very famous for his book *The Structure of Scientific Revolutions*; Kolb is an important cosmologist. For a broader history of astronomy and cosmology, see Timothy Ferris's *Coming of Age in the Milky Way.* Copernicus has also received a new biography from William T. Vollmann, *Uncentering the Earth: Copernicus and* The Revolutions of the Heavenly Spheres. *De Revolutionibus* is available in an English translation but it's hard going.

Only in the last twenty-five years or so have most of Kepler's works finally been translated into English. The most important, *New Astronomy* appeared in 1992, translated by William Donahue, who has also translated Kepler's *Optics.* Recently, Kepler's charming and speculative essay *On the Six-Sided Snowflake* has been republished in paperback. Kepler is also among the earliest of science fiction writers; his *Somnium* is a dream of flight to the moon.

For Galileo references, see §§12 & 14.

The literature on Newton is enormous. Start with the elegant, brief biography by James Gleick, *Isaac Newton.* For Newton's work on gravity, it is hard to imagine a better guide than Dana Densmore's *Newton's Principia: The Central Argument.* The greatest

[82] A collection of Newton's papers held by Conduitt's descendants was auctioned off by Sotheby's in 1936. Many were purchased by the eminent economist John Maynard Keynes. Conduitt's record of his conversation with de Moivre was bought by Joseph Halle Schaffner, a founder of the men's clothing firm of Hart Schaffner Marx, and willed to the University of Chicago's Regenstein Library, where it may be seen today.

modern scholars on Newton were I. Bernard Cohen of Harvard and D. T. (Tom) Whiteside of Cambridge. If you ever need to learn some small fact about Newton or his work, consult their books first. W. W. Rouse Ball's *Essay on Newton's Principia* is still worth reading a century after it was written; much of Newton's correspondence with Hooke and Halley is reprinted there.

There is a recent biography of Edmond Halley by Alan H. Cook, *Edmond Halley: Charting the Heavens and the Seas.* In 1705 Halley would predict the return of a comet in 1758, which he thought was the same object pictured on the Bayeux Tapestry of 1066. According to his investigations, a comet had made regular visits about every 76 years, and would continue to do so. Halley was proved correct posthumously, and the comet became Halley's. But all that was years away when Halley met Hooke and Wren to talk about gravity. Halley was also a pioneer of actuarial science (the mathematics of insurance policies) and provided an important translation from Arabic (see the notes to §32.) Incidentally, Mark Twain was born during a visit of Halley's comet. In 1909 Twain said

> I came in with Halley's Comet in 1835. It is coming again next year, and I expect to go out with it. It will be the greatest disappointment of my life if I don't go out with Halley's Comet. The Almighty has said, no doubt: "Now here are these two unaccountable freaks; they came in together, they must go out together."

Right on schedule, Twain died the day after the comet's closest approach to the earth, 21 April 1910.

Lisa Jardine has written recent biographies of both Robert Hooke (*The Curious Life of Robert Hooke*) and Christopher Wren (*On A Grander Scale*). Wren deserves further mention. The grandest building in London is probably St. Paul's Cathedral, which Wren designed after St. Peter's in Rome, on the foundations of an earlier church by that name destroyed in the Great Fire. If you visit London, and if you have a head for heights, you can walk to the top of the dome and then outside of it for an unequalled aerial view of the great city. Wren is buried in the crypt of this majestic church, his *magnum opus.* It's impossible to resist quoting his epitaph, composed by his son in 1723:

Subtus conditur huius ecclesiae et urbis conditor qui vixit annos ultra nonaginta non sibi sed bono publico.	Beneath is he found, the founder of this church and the city who lived past ninety years not for himself but for the public good.
Lector, si monumentum requiris, circumspici.	Reader, if you seek a monument, look around you.

§31. The System of the World

Many advances in physics have come from an almost reckless generalization of a conclusion drawn from the study of one system to a law of nature. Einstein showed that light's energy E had an effective mass of E/c^2, and deduced from that the equivalence of *all* forms of energy and mass. In 1923, Louis de Broglie (1892-1987) suggested in his Ph.D. dissertation that if light could act as both particles and waves, then perhaps so could electrons, and maybe all matter. De Broglie might have been failed for his boldness, but for one examiner, Paul Langevin (1872-1946), who asked his friend Einstein to come to the student's defense. With Einstein's approval, De Broglie's work was found to be good enough for his degree; three years later, it was good enough to jump start quantum mechanics, and six years later, good enough for the Nobel. The first great generalization is Newton's formulation of gravity. How did he do it? Here is one possible route. It may not be the way he did it, but it could have been.

Pretend that all the planetary orbits are circular. (This isn't strictly necessary, but it makes the math much easier. And it is not far from the truth.) Let some planet P have a mass m_P. With the planet in a circular orbit,

$$F_{\text{net on } P} = \frac{m_P v^2}{r} = \frac{m_P(2\pi r/T)^2}{r} = \frac{4\pi^2 m_P r}{T^2}$$

because for circular motion, $v = 2\pi r/T$. From Kepler's Third Law,

$$\frac{r^3}{T^2} = K \Rightarrow \frac{1}{T^2} = \frac{K}{r^3} = \frac{K_S}{r^3}$$

where a subscript S is put on the Kepler constant, K, because, after all, that is what the planets have in common; they orbit the Sun. Substituting this into the formula for the net force gives

$$F_{\text{net on } P} = \frac{4\pi^2 m_P r}{T^2} = \frac{4\pi^2 K_S m_P}{r^2}$$

This force comes, presumably, from the Sun acting on the planet;

$$F_{S \text{ on } P} = \frac{4\pi^2 K_S m_P}{r^2} \qquad (*)$$